

**DEVELOPING MATHEMATICAL DISCOURSE - IN CLASS AND BEYOND: A COMMUNICATIONAL PERSPECTIVE**Chair: **A. Sfard**<sup>1</sup>**B. Schwarz**<sup>2</sup>, **E. Heyd-Metzuyanim**<sup>3</sup>, **S. Caspi**<sup>4</sup>, **M. Tabach**<sup>5</sup>,  
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**INTRODUCTION: DEVELOPING MATHEMATICAL DISCOURSE  
- SOME INSIGHTS FROM COMMUNICATIVE  
(COMMUNICATIONAL) RESEARCH**

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Although all presentations in this symposium speak about mathematics and learning, they may appear too diverse in their foci to be bound together. The issues to be discussed in this symposium, when taken together, cover at least three separate research areas, those of cognition, affect, and social interactions. Traditionally, these three types of study differ from each other not just in their themes, but also in their foundational assumptions and methods. One of the main aims of this symposium is to break out from the grip of the separatist tradition, the tradition of using different, often unbridgeable discourses for dealing with different aspects of learning. The importance of the project of bridging and unifying can hardly be overestimated. If the collective effort of those who study learning-teaching processes is to result in a picture of the proverbial elephant rather than in a collection of possibly misleading partial images, researchers need to build on each other's work; to be able to do so, they have to communicate with one another; and in order to communicate, they need a common discourse, one in which cognitive and affective, as well as intra-personal and inter-personal (or individual and social) aspects of teaching-learning processes.

## **SPONTANEOUS META-ARITHMETIC AS A THE FIRST STEP TOWARD SCHOOL ALGEBRA**

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Taking as the point of departure the vision of school algebra as a formalized meta-discourse of arithmetic, I have been following five pairs of 7<sup>th</sup> grade students as they progress in algebraic discourse during 24 months, from their informal algebraic talk to the formal algebraic discourse, as taught in school. My analysis follows changes that occur along time in the discourse produced by the students when they deal with specific types of algebraic problems.

The participants have been interviewed at the intervals of 6 months. Each round of interviewing consisted of 5 meetings lasting for 60 minutes. The first round began just before the students were introduced to algebra in school.

My data have shown, unsurprisingly, that while reflecting on arithmetic processes and relations, the uninitiated 7<sup>th</sup> graders were employing colloquial means. More unexpectedly, this spontaneous meta-arithmetic, although not supported by any previous algebraic schooling, displayed some algebra-like features, not to be normally found in everyday discourses. This finding prompted me to extend the study and take a look at spontaneous meta-arithmetic of younger children. As a result of the initial analyses carried out after the first round of interviewing and realizing that I need additional data in order to choose from several alternative interpretations of the data collected that far, I decided on an *ad hoc* extension of the project. The additional interviews were conducted with children younger than our original cohort. My data collected among 5<sup>th</sup> graders have confirmed that informal algebraic discourse emerges early, and they shed light on the development of meta-arithmetic in the span of two years prior to the formal introduction of algebra in school.

In my talk, I will describe these two moments in the initial phases of my ongoing study of this topic.

**THE CO-CONSTRUCTION OF 'LEARNING DIFFICULTIES' IN MATHEMATICS: TEACHER-STUDENT INTERACTIONS AND THEIR ROLE IN THE DEVELOPMENT OF A 'DISABLED' MATHEMATICAL IDENTITY**

**E. Heyd-Metzuyanin**

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The communicational (commognitive) lens was in the present study for analysing processes of identity construction which take place between a teacher and a student in one-on-one interactions and in class. The case reported here was part of a larger study which was aimed at developing a method for studying emotional, social and cognitive aspects of mathematical learning using a unified set of conceptual tools. The basic conceptual division made within this method of discourse analysis is between "mathematizing" (communicating about mathematical objects) and "subjectifying" (communicating about the *participants* of the discourse). Subjectifying utterances are categorized according to their "identifying" content – the extent to which they refer to stable properties of a person and thus construct his/her identity. Here, these conceptual tools were used to analyse the case of Dana, who was the lowest achieving of 12 7<sup>th</sup> grade students whom I taught over a period of 5 months in an out-of-school program. During these months, Dana failed to advance in any of her mathematical skills despite intensive instruction. Close analysis of her forms of participation in my lessons revealed that this lack of advancement could be traced to the ritual rule following and "direct instructions" in which she and I engaged, which contradicted my initial attempts to introduce a more "dialogic" form of teaching and learning. Analysis of the *interactional routines* which Dana and I followed revealed that Dana repeatedly deviated from normative routines when given a chance to participate in explorative discussion, specifically in their subjectifying content, leading me to identify her as "clueless". My story of Dana, in turn, led me to stick to limited procedural ("direct") instruction. Thus both Dana and I participated in constructing her identity as "disabled" in mathematics, and simultaneously constrained her forms of participation to be ineffective for her mathematical advancement.

**DEVELOPING THE MATHEMATICAL OBJECT FUNCTION:  
THE CASE OF PROSPECTIVE TEACHERS<sup>1</sup>****M. Tabach<sup>1</sup>, T. Nachlieli<sup>2</sup>**<sup>1</sup>Tel Aviv University; <sup>2</sup>Levinsky College of Education, Tel Aviv, Israel

The paper presented in this talk focuses on learning processes of prospective elementary school mathematics teachers who are studying a course on functions. Specifically, two questions are addressed – *how did prospective mathematics teachers further develop the object of function?* and *how did the prospective teachers' and their instructor's actions enable and promote this development?* For this purpose, a one-semester course on functions was videotaped and transcribed. The mathematics classroom discourse that developed in class was analyzed using the commognitive framework, by considering the four characteristics of discourse: word-use, visual mediators, endorsed narratives and routines. We noticed three different discourses that merge simultaneously in class – the mathematical, the interpersonal and the pedagogical. In this talk we will speak about our choice of defining those discourses. Our findings suggest that the interpersonal and the pedagogical discourses enabled and promoted the development of students' mathematical discourse about functions. Specifically, the object function was developed both in the object level (object level learning includes expanding the existing discourse) and in the meta-level (meta level learning includes introducing new objects and changing meta-rules of the discourse). These findings contradict results obtained in previous studies about the use of definitions in general, and particularly their use to identify function. Yet, our findings suggest that not all function mediators were developed to the same extent. Graphs, for example, were developed more than symbolic expression.

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